HOMEWORK FOR MATH 147 FALL 2025

Problems assigned from Marsden and Weinstein will be preceded by MW. Problems assigned from the Openstax Calculus 3 textbook will be preceded by OS.

Monday, August 18. MW: Section 14.3, #3, 5, 13, 15, 37, 43.

Wednesday, August 20. OS: Section 4.2, # 61, 62, 63, 66, 74, 80, 81.

Friday, August 22. 1. Use polar coordinates to analyze the following limits:

(i) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$.

(ii) $\lim_{(x,y)\to(0,0)} \frac{x^3+x^5}{x^2+y^2}$.

2. Determine the value of the constant c so that $f(x,y) = \begin{cases} \frac{x^3+xy^2+2x^2+2y^2}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ c, & \text{if } (x,y) = (0,0) \end{cases}$ is a continuous function.

3. OS: Section 4.2, $\#102, 110, 111$.

4. For $F(x,y,z) = (x^2+y^2+z^2, 3xyz, \cos(x)+\sin(y)+e^z)$, calculate $\lim_{(x,y,z)\to(1,-1,1)} F(x,y,z)$.

Monday, August 25. MW, Section 15.1: # 13-41, every other odd problem, and # 53.

2. For $f(x,y) = \begin{cases} \frac{3x^2y-y^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0), \end{cases}$ find formulas for $f_x(x,y)$ and $f_y(x,y)$. Note: You can just take partial derivatives as usual when $(x,y) \neq (0,0)$, and then use the limit definition to find what the partial derivatives are when $(x,y) = (0,0)$.

Wednesday, August 27. OS, Section 4.4: Find the tangent lines in the x and y directions for the functions and points given in # 171, 173, 176,. Then try to find the tangent planes, using the corresponding tangent vectors. Also: Use the limit definition to show that $f(x,y) = 3x^2 + y$ is differentiable at (1, -1). Then try showing $f(x,y)$ is differentiable at any point (a,b).

Friday, August 29. OS, Section 4.4: # 179, 191 and: (i) Use the definition of differentiability to show that $2x^2+3y$ is differentiable at all $(a,b) \in \mathbb{R}^2$ and (ii) Determine whether or not the function

$f(x,y) = \begin{cases} \frac{x^5}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$ is differentiable at (0,0).

Wednesday, September 3. 1. For the function $f(x,y) = \begin{cases} \frac{2x^2y^2}{\sqrt{x^2+y^2}}, & \text{if } (x,y) = (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$, use the definition to show that $f(x,y)$ is differentiable at (0,0). Then verify that both partial derivatives are continuous at (0,0).

2. Use the definition to verify that $f(x,y,z) = xyz + 75$ is differentiable at all points $(a,b,c) \in \mathbb{R}^3$.

Friday, September 5. Find $DF(2,3,1)$ for the function $F(x,y,z) = (x^2y^3z, e^{xy^2z^3}, \cos(xyz))$ and OS, Section 4.7: # 311-339, every other odd. Just find the critical points, **don't classify them**.

In addition, use an ϵ, δ argument to show that, given a function $f(x,y)$, a point (a,b) in its domain, and $L \in \mathbb{R}$, the statements $\lim_{(x,y)\to(a,b)} f(x,y) = L$ and $\lim_{(x,y)\to(a,b)} |f(x,y) - L| = 0$ are equivalent, i.e., each statement implies the other statement.

Monday, September 8. OS, Section 4.7: Classify the critical points you found in # 319-339, every other odd, in the previous assignment.

Wednesday, September 10. MW, Section 16.3: # 21, 24, 27, 32, 34, and OS, Section 4.7, # 346, 347, 348.

Friday, September 12. 1. Find and classify the critical points for: $f(x, y, z) = x^2 - xy + z^2 - 2xz + 6z$ and $g(x, y, z) = xy + xz + 2yz + \frac{1}{x}$.

2. OS, Section 4.5: #215, 217, 219, 243, 244, 254.

Monday, September 15. MW, Section 16.1: # 21, 22, 27, 33; And: Use the limit definition to find the directional derivative of $f(x, y) = 3x^2 + 2xy + 5$ at (1,2) in the direction of $\cos(\frac{\pi}{3})\vec{i} + \sin(\frac{\pi}{3})\vec{j}$, then verify your answer using the gradient formula..

Wednesday, September 17. This homework problem is **Bonus Problem 4**, to be turned in on Friday, September 19 for a maximum of three bonus points. In class we noted that iterated limits need not be equal, for functions of two variables. The failure of the equality of the limits $\lim_{k\to a} \lim_{h\to b} L(h, k)$ and $\lim_{h\to b} \lim_{k\to a} L(h, k)$ for $L(h, k) = \frac{h+k}{h-k}$, is related to the failure of the $\lim_{(h,k)\to(0,0} L(h, k)$ to exist. Here is a sufficient condition:

Equality of Iterated Limits. Given $f(x, y)$ and $(a, b) \in \mathbb{R}^2$, If

(i) $\lim_{(x,y)\to(a,b)} f(x, y)$ exits, and
(ii) $\lim_{x\to a} f(x, y)$ exists for fixed y, and
(iii) $\lim_{y\to b} f(x, y)$, exists for fixed x,

then $\lim_{x\to a} \lim_{y\to b} f(x, y) = \lim_{(x,y)\to(a,b)} f(x, y) = \lim_{y\to b} \lim_{x\to a} f(x, y)$.

1. For $f(x, y) = \frac{x^2}{x^2+y^2}$, show that $\lim_{(x,y)\to(0,0)} f(x, y)$ does not exist, while each of $\lim_{y\to 0} \lim_{x\to 0} f(x, y)$ and $\lim_{x\to 0} \lim_{y\to 0} f(x, y)$ exist, but are not equal.

2. For $f(x, y) = \frac{x^2+y+1}{x+y^2+1}$, show that $\lim_{(x,y)\to(0,0)} f(x, y)$, $\lim_{y\to 0} \lim_{x\to 0} f(x, y)$, $\lim_{x\to 0} \lim_{y\to 0} f(x, y)$ exist and are all equal.

3. For $f(x, y) = \begin{cases} 1, & \text{if } xy \neq 0 \\ 0, & \text{if } xy = 0 \end{cases}$ show that $\lim_{y\to 0} \lim_{x\to 0} f(x, y) = 1 = \lim_{x\to 0} \lim_{y\to 0} f(x, y)$, but $\lim_{(x,y)\to(0,0)} f(x, y)$ does not exist.

Wednesday, September 24. OS, Section 4.8: # 361-366.

Bonus Problem 5. Let S be the surface that is the graph of the equation $z = f(x, y)$ and suppose that $P = (a, b, f(a, b))$ is a point on S. Let L_0 be a line in $\mathbb{R}^2$ passing through (a, b) and C denote the curve consisting of the points on S lying above L_0. Let $\vec{u} = u_1\vec{i} + u_2\vec{j}$ be a unit direction vector for L_0. Give a rigorous explanation for why

$$L(t) = (a, b, f(a, b)) + t(u_1, u_2, D_{\vec{u}}f(a, b))$$

is the parametric equation of the line tangent to C at the point P. We will assume that $f(x, y) \geq 0$ in an open disk about (a, b) (so the surface lies above the xy-plane near P) and the first order partials of $f(x, y)$ exist and are continuous in an open disk about (a, b). Due Friday, September 26. (4 points)

Friday, September 26. OS, Section 4.8: # 377, 379, 382, 384, 387

Monday, September 29. OS, Section 5.1: # 13, 19, 21, 25, 37, 30.

Wednesday, October 1. MW, Section 17.2: # 7-19, odd.

Bonus Problem 6. Work the following problem for three bonus points and turn in your solution on Friday, October 3. Suppose $a(t)$ is a function of one variable, and $f(x, y) = a(x)a(y)$. Let R denote the square $[c, d] \times [c, d]$. Prove that $\int\int_R f(x, y)\, dA = (\int_c^d a(x)\, dx)^2$.

Friday, October 3. OS, Section 5.3: # 149,154,155,158, 159.

Monday, October 6. OS, Section 5.7: # 388, 389, 392, 398.

Wednesday, October 8. OS, Section 5.7: # 390, 394, 397.

Bonus Problem 7. Suppose $T(u, v) = (au + bv, cu + dv)$ is a linear transformation from the uv-plane to the xy-plane. Give a good proof that T is one-to-one if and only if $ad - bc$ is not zero. This problem is due in class on Wednesday October 15 and is worth 5 points. Hint: For one direction, you will end up solving a system of two homogeneous equations in two unknowns.

Friday, October 10. OS, Section 5.6: # 391, 396. Note that these problems give the inverse transformation.

Wednesday, October 15. Calculate the following improper integrals.

(i) $\int\int_D \frac{1}{\sqrt{xy}}\, dA$, for $D = [0,1] \times [0,1]$.

(ii) $\int\int_D \ln\sqrt{x^2+y^2}\, dA$, for $D = 0 \le x^2+y^2 \le 1$.

(iii) $\int\int_D \frac{1}{x^2y^3}\, dA$, for $D = [1,\infty] \times [1,\infty]$.

Friday, October 17. MW, Section 17.4: # 1, 5, 9, 16.

Monday, October 20. OS, Section 5.4: #193, 194, 211, 212, 231.

Wednesday, October 22. Section 5.5: # 253-256, 269, 270, 271.

Wednesday, October 29. 1. Calculate $\int\int\int_B 3x+y+z^2\, dV$ where B is the solid parallelopiped spanned by the vectors $v_1 = (1,1,1), v_2 = (1,2,1), v_3 = (2,2,2)$.

2. Let B_0 denote the solid cube in the uvw-coordinate system centered at the origin with sides of length 2. Let B denote the solid box in the xyz-coordinate system centered at (1,-2,1) whose sides have lengths 2, 4, 6 in the x, y, z directsions. Find a transformation $G(u,v,w)$ from the (u,v,w)-coordinate system to the xyz-coordinate system taking B_0 to B.

3. For B as in problem 2 above, use the transformation you found to calculate $\int\int\int_B xyz\, dV$.

Bonus Problem 8. To be turned in Friday during class.

For a 3×3 matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, define A_{ij}, for $i \neq j$, to be the 2×2 matrix obtained by deleting the ith row and jth column of A. We can define the determinant of A by expanding along any row or any column, according to the following formulas. In the formulas below, we use $|C|$ to denote the determinant of the matrix C, so that, in the present situation, $|\ -\ |$ does not mean absolute value.

$$\begin{aligned} |A| &= \sum_{j=1}^{3}(-1)^{i+j}a_{ij}\cdot|A_{ij}|, \quad \text{expansion along the } ith \text{ row} \\ &= \sum_{i=1}^{3}(-1)^{i+j}a_{ij}\cdot|A_{ij}|, \quad \text{expansion along the } jth \text{ column.} \end{aligned}$$

Now let A denote the matrix $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$.

1. Use the formulas above to show that $|A|$ is the same when expanding along the third row or expanding along the second column. (2 points)

3. Show that $|A| = |A^t|$, where A^t denoted the transpose of A, i.e., $A^t = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$. (3 points)

Friday, October 31. OS, Section 3.2:# 41-55, odd.

Bonus Problem 9. Give a proof of the differentiablity properties (4)-(6) from today's lecture. Each part is worth 2 points, and this is due Monday, November 3.

Monday, November 3. OS, Section 3.3: # 102, 106, 107, 110 and MW, Section 18.1: # 29, 30, 32, 34.

Wednesday, November 5. OS, Section 6.2: # 75, 86, 92, 94 and the following problem: Let C be the curve with parametrization $\mathbf{r}(t) = (\cos(t), \sin(t), t)$, $0 \le t \le 2\pi$ so that C is that portion of the helix of radius one from (1,0,0) to (1,0,1). Find a second parametrization of C and use this to create a re-parametrization of C. Then check that $\int_C x+y+z\, ds$ is independent of the two parameterizations.

Friday, November 7. 1. For the sphere $S: x^2+y^2+z^2=4$, find the plane tangent to S at $P=(1,1,\sqrt{2})$.

2. Let S denote the surface that is the graph of the function $z=f(x,y)$. In terms of x,y,z, find the equation of the plane tangent to S at the point $P=(x_0,y_0,z_0)$.

3. Find a parameterization in terms of u,v for the plane you found in problem 1.

Bonus Problem 10. Look up in any calculus book the definition of of $\vec{v}\times\vec{w}$ for vectors $\vec{v},\vec{w}\in\mathbb{R}^3$. Read and then write down proofs of the following facts: (a) $\vec{v}\times\vec{w}$ is orthogonal to the plane spanned by $\vec{v}$ and $\vec{w}$ (assuming these vectors are not collinear) and (b) The length of $\vec{v}\times\vec{w}$ equals the area of the parallelogram spanned by $\vec{v}$ and $\vec{w}$. This problem is worth 5 points and is due in class on Monday, November 10.

Monday, November 10. 1. Calculate $\int\int_X \sqrt{x^2+y^2+1}\, dS$, where S is the helicoid given parametrically by $G(r,\theta)=(r\cos\theta, r\sin\theta,\theta)$, with $0\le r\le 1$ and $0\le\theta\le 2\pi$. What is the surface area of S?

2. Let S denote the unit cube in the first of $\mathbb{R}^3$ spanned by $\vec{e},\vec{e_e},\vec{e_3}$. Calculate $\int\int_S xyz\, dS$. Hint: There are six separate surface integrals to calculate, but three of them have an obvious answer (with a little thought).

Wednesday, November 12. OS Section 6.2: # 68, 69, 70 and OS Section 6.6: # 303, 304, 305, 309.

Friday, November 14. 1. Suppose $\mathbf{F}=x\vec{i}+y\vec{j}+(z-2)\vec{k}$. Calculate $\int\int_S \mathbf{F}\cdot d\mathbf{S}$, for S the helicoid with parameterization $G(u,v)=(u\cos(v), u\sin(v), v)$, with $0\le u\le 1$ and $0\le v\le 2\pi$.

2. Let $\mathbf{F}=x\vec{i}+y\vec{j}+z\vec{k}$ and S denote the sphere of radius R centered at the origin. Calculate $\int\int_S \mathbf{F}\cdot d\mathbf{S}$ with respect to the outward normal in two ways: First by parameterizing S and second, without parameterizing S, i.e., by just thinking about the situation.

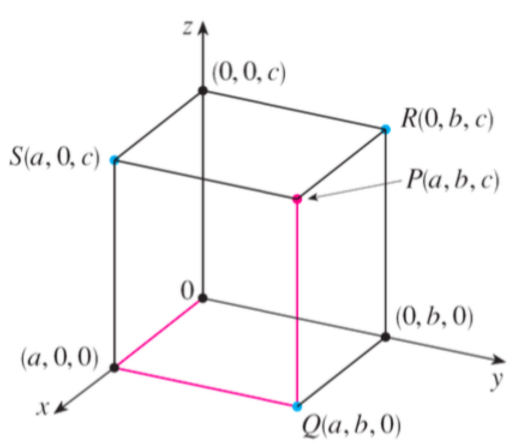
3. Suppose S is the graph of $z=f(x,y)$ defined over $D\subseteq\mathbb{R}^2$. Let $h(x,y,z)$ be a scalar function defined on S and $\mathbf{F}=F_1(x,y,z)\vec{i}+F_2(x,y,z)\vec{j}+F_3(x,y,z)\vec{k}$ be a vector field defined on S. Show that:

(i) $\int\int_S h(x,y,z)\, dS=\int\int_D h(x,y,f(x,y))\sqrt{1+f_x(x,y)^2+f_y(x,y)^2}\, dxdy$.

(ii) $\int\int_S \mathbf{F}\cdot d\mathbf{S}=\int\int_D -F_1(x,y,f(x,y))f_x(x,y)-F_2(x,y,f(x,y))f_y(x,y)+F_3(x,y,f(x,y))\, dxdy$.

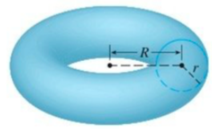
Hint: Use x,y to parameterize S.

Monday, November 17. Verify the Divergence Theorem for $\mathbf{F}=x^2i+y^2j+z^2k$, and B the solid rectangle $0\le x\le a$, $0\le y\le b$, $0\le z\le c$.



Wednesday, November 19. 1. Calculate $\int\int_S \mathbf{F}\cdot d\mathbf{S}$ with respect to the outward normal, for the vector field $\mathbf{F}=yz^3\vec{i}+e^{x^2+z^2}\vec{j}+\cos(\sqrt{x^2+y^2})\vec{k}$ and S torus obtained by revolving the circle $(y-3)^2+z^2=4$ in the

yz-plane about the y-axis,



A parametrization for S is: $x = (3+2\cos(v))\sin(u)$, $y = (3+2\cos(v))\cos(u)$, $z = 2\sin(v)$, with $0 \le u, v \le 2\pi$.

2. Verify the Divergence Theorem for $\mathbf{F} = (x-y)\vec{i} + (x+z)\vec{j} + (z-y)\vec{k}$, for the surface that consists of the cone $x^2 + y^2 = z^2$, $0 \le z \le 1$ with a circular top at the $z = 1$ level.

Thursday, November 20. 1. Verify Green's Theorem for $\mathbf{F} = (x^2y + x)\vec{i} + (y^3 - xy^2)\vec{j}$ and D the region bounded by the the circles $x^2 + y^2 = 9$ and $x^2 + y^2 = 4$. Note that ∂D has an inner component and outer component. These must be oriented correctly so that the region D remains on the left as one travels along ∂D.

2. Use a line integral to find the area contained in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Friday, November 21. Set $\mathbf{F} = z^2\vec{i} + x^2\vec{j} - y^2\vec{k}$.

(i) Calculate $\nabla \times \mathbf{F}$.
(ii) Let C be the square path with sides equal to a centered at the point $(x_0, 0, z_0)$ lying in the xz-plane oriented so that each side is parallel to the x or z axis. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.
(iii) Divide your answer in (ii) by the area of the square and take the limit as a goes to zero.
(iv) Use your answer in (i) to corroborate your answer in (iii).

2. Verify Stoke's Theorem for $\mathbf{F} = (-y, 2x, x+z)$ and S the upper hemisphere of the sphere of radius R centered at the origin.

Monday, November 24. OS, section 6.7: #335, 339, 343.